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Transverse Emittance Growth During γ_T Jump *

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TRANSVERSE EMITTANCE GROWTH DURING γ_T JUMP

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I. INTRODUCTION

Acceleration of a particle beam across transition will usually lead to an increase in the bunch area and a loss in beam intensity. This is because (1) different particle crosses transition at different time leaving behind two tails in the longitudinal phase space, and (2) the beam becomes microwave unstable when the phase-slip factor $\eta = 1/\gamma_T^2 - 1/\gamma^2$ is negligibly small. Tracking simulation[1] indicates that particle loss will be about 20% and the bunch area growth can be a factor of $2 \sim 3$ for the proposed Fermilab Main Injector. These problems can be avoided by implementing a γ_T jump system, which consists of pulsing quadrupole magnets to change the optics of the accelerator in such a way as to drop γ_T of the machine *instantly* as the beam approaches transition. Under this situation, the particles cross transition so fast that none of the nuisances mentioned above would have time to develop. However, changing the optics of the ring instantly will lead to a sudden growth in the transverse emittance of the beam, which is certainly undesired. To preserve the transverse emittance, the γ_T jump has to be performed adiabatically. It is the purpose of this paper to find out the shortest time of the jump so that the transverse emittance will not be disturbed significantly.

II. THE MODEL

Consider the particle in the bunch which has the largest fractional energy offset δ (corresponding to 95% bunch area). This particle has a maximum transverse offset of $X_p^i\delta$ from the synchronous orbit, where X_p^i is the maximum momentum dispersion of the ring. Now for the performance of γ_T jump, the quadrupoles are pulsed so that the maximum dispersion changes to the final value of X_p^f in n turns. Suppose that the

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dispersion is changed evenly as $k = (X_p^f - X_p^i)/n$ per turn. Therefore, this particle is performing betatron oscillations about a different closed orbit in a different turn.

The equation of motion of x, the transverse displacement of the particle from the synchronous orbit, is, in our simplified model,

$$x'' + K(s)(x - X_p\delta) = 0 \ , \tag{2.1}$$

where the prime is differentiation with respect to s, the distance measured along the synchronous orbit from some reference point where the dispersion is a maximum and the betatron function β is a local maximum, $K(s)$ is the quadrupole strength, and

$$X_p = X_p^i + mk \ , \tag{2.2}$$

is the our assumed dispersion for the mth turn. This is the model of of an oscillator driven by a force $K(s)X_p\delta$, which changes abruptly whenever the particle passes through the reference point at every turn. The particle position x as well as the angle x' are therefore continuous at the reference point. This model is reasonable because the pulsing of the quadrupoles during the γ_T jump is performed in such a way that the phase advance and tune of the accelerator ring are essentially unchanged.

According to the equation of motion, the transverse position of the particle at the mth turn relative to the synchronous orbit is

$$x_m = (X_p^i + mk)\delta + a_m \cos\psi + b_m \sin\psi \qquad m = 0,\ 1,\ 2,\ \cdots\ , \tag{2.3}$$

where ψ is the Floquet phase advance along the synchronous orbit measured from some reference point. For every revolution around the ring ψ increases by $\mu = 2\pi\nu$, where ν is the betatron tune. For convenience, we set $\psi = 0$ at the beginning of every turn. In general a_n and b_n are proportional to the square root of the betatron function β along the ring. However, since we are interested in only the reference point, we have

$$\begin{aligned} x_m{}^- &= (X_p^i + mk)\delta + a_m && \text{beginning of } m\text{th turn,} \\ x_m{}^+ &= (X_p^i + mk)\delta + a_m \cos\mu + b_m \sin\mu && \text{end of } m\text{th turn,} \end{aligned} \tag{2.4}$$

and

$$\begin{aligned} x'_m{}^- &= \frac{b_m}{\beta} && \text{beginning of } m\text{th turn,} \\ x'_m{}^+ &= -\frac{a_m}{\beta}\sin\mu + \frac{b_m}{\beta}\cos\mu && \text{end of } m\text{th turn.} \end{aligned} \tag{2.5}$$

Here, a_m and b_m/β represent the betatron oscillation amplitude and angular deviation at the reference point. The emittance of this off-energy particle is therefore given by

$$\epsilon = \pi\gamma\frac{r_m^2}{\beta} \qquad \text{with} \qquad r_m = \sqrt{a_m^2 + b_m^2} \ . \tag{2.6}$$

From the continuation of x_m and x'_m across the reference point, we obtain

$$\begin{aligned} a_m &= -k\delta + a_{m-1}\cos\mu + b_{m-1}\sin\mu \ , \\ b_m &= -a_{m-1}\sin\mu + b_{m-1}\cos\mu \ . \end{aligned} \tag{2.7}$$

The eventual maximum transverse displacement of the particle will be given by

$$x_{\max} = X_p^f\delta + r_n \ , \tag{2.8}$$

where r_n is the betatron amplitude at the nth turn.

III. SPECIAL CASES

1. Integer tune

If the tune ν is an integer, the particle returns to its original position after each turn, although the off-energy closed orbit is altered every turn. Thus the betatron amplitude is always equal to $mk\delta$ in the mth turn, if we start with $a_0 = b_0 = 0$. The final transverse displacement is therefore

$$x_{\max} = X_p^f\delta + nk\delta = (2X_p^f + X_p^i)\delta \ , \tag{3.1}$$

which is the same as having the quadrupole pulsed to the final value in only one turn.

2. Half-integer tune

Starting from $a_0 = b_0 = 0$ and a half-integer tune, it is obvious that $b_n = 0$. Since the off-energy closed orbit changes by the same amount every turn, the displacement of the particle can be followed easily. The results for the first 7 turns are listed in Table I. We see that the amplitude of betatron oscillation r_n is either zero or one unit of the shift of the off-energy closed orbit $k\delta$ depending on whether the turn number is even or odd. Therefore, the maximum transverse displacement of the beam is

$$x_n = \left[X_p^f + \frac{1}{n}(X_p^f - X_p^i)\right]\delta \ . \tag{3.2}$$

Turn No.	Dispersion Change in k	a_n in $k\delta$	
		beginning	end
0	0	0	0
1	1	-1	1
2	2	0	0
3	3	-1	1
4	4	0	0
5	5	-1	1
6	6	0	0

Table I: Betatron oscillation amplitude for the first 7 turns at half-integer tune

IV. GENERAL SOLUTION

With any other tunes, the betatron oscillation of the particle is in between the worst scenario of integer tune and the best scenario of half-integer tune. The betatron oscillation amplitude of the off-energy particle can be computed from Eqs. (2.7), which we rewrite as

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = O \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} - \begin{pmatrix} 0 \\ k\delta \end{pmatrix} , \tag{4.1}$$

where

$$O = \begin{pmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{pmatrix} \tag{4.2}$$

is the transportation matrix of one revolution around the ring from the quadrupole back to the quadrupole or just the rotation matrix of an angle μ. We can easily iterate Eq. (4.1) to give

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = O^n \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} - \sum_{m=1}^{n} O^{n-m} \begin{pmatrix} 0 \\ k\delta \end{pmatrix} . \tag{4.3}$$

If the initial betatron oscillation is negligibly small or $a_0 \sim 0$, $b_0 \sim 0$, Eq. (4.3) reduces to

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = -k\delta \sum_{m=1}^{n} \begin{pmatrix} \cos(n-m)\mu \\ \sin(n-m)\mu \end{pmatrix} , \tag{4.4}$$

which can be summed easily to give

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = -k\delta \begin{pmatrix} \mathcal{Re} \\ \mathcal{Im} \end{pmatrix} e^{-i\frac{n-1}{2}\mu} \frac{\sin\frac{n}{2}\mu}{\sin\frac{1}{2}\mu} , \tag{4.5}$$

or

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = -k\delta \begin{pmatrix} \dfrac{\sin\frac{n}{2}\mu\cos\frac{n-1}{2}\mu}{\sin\frac{1}{2}\mu} \\ \\ \dfrac{\sin\frac{n}{2}\mu\cos\frac{n-1}{2}\mu}{\sin\frac{1}{2}\mu} \end{pmatrix} . \tag{4.6}$$

The betatron oscillation amplitude is therefore

$$r_n = \frac{\Delta X_p \delta}{n} \left| \frac{\sin\frac{n}{2}\mu}{\sin\frac{1}{2}\mu} \right| , \tag{4.7}$$

where we have made the substitution $k = \Delta X_p/n = (X_p^f - X_p^i)/n$.

It is clear from Eq. (4.7) that when the betatron tune is an integer, nothing can be gained by pulsing the quadrupoles adiabatically, as was pointed out in Section III. When the tune is of half-integer, the result of Section III.2 is reproduced.

As is shown in Eq. (4.7), r_n is very sensitive to the tune. For example, if the residual tune $[\nu] = 0.25$, r_n vanishes exactly whenever n is a multiple of 4. This peculiar result comes about because a_0 and b_0 are not exactly zero to begin with and that our model, Eq. (2.1), has been too simple. In order to obtain a more meaningful result, we replaced $|\sin n\mu/2|$ by its maximum value unity, except when the tune is very near to an integer. Then the betatron amplitude becomes

$$r_n = \frac{\Delta X_p \delta}{n} \left| \frac{1}{\sin\frac{1}{2}\mu} \right| . \tag{4.8}$$

We see that the last factor is roughly less than 2 when the residual tune is between 0.15 and 0.85.

V. ADIABATIC CRITERION

We would like the final betatron oscillation much less than the initial betatron oscillation. Therefore, the adiabatic criterion is

$$\frac{\Delta X_p \delta}{n} \frac{1}{\left|\sin \frac{1}{2}\mu\right|} \ll A_\beta \ , \tag{5.1}$$

where A_β is the amplitude of betatron oscillation at maximum β before transition, which is related to the normalized emittance ϵ by

$$\epsilon = \pi\gamma \frac{A_\beta^2}{\beta} \ . \tag{5.2}$$

For the Main Injector, $\epsilon = 20\pi$ mm-mr (95%), maximum $\beta = 57$ m, and $\gamma_T = 20.4$. Therefore $A_\beta = 0.00748$ m. One proposal[2] of γ_T jump boosts the momentum dispersion according to

$$X_p = 2.2 + 4.8(\Delta\gamma_T)^{\frac{1}{2}} \text{ m}, \tag{5.3}$$

or $\Delta X_p = X_p^f - X_p^i = 4.8$ m for $\Delta\gamma_T = 1$. The fractional momentum spread is

$$\delta = 6.00 \times 10^{-3} \Delta\gamma_T^{-\frac{1}{4}} \left(\frac{S}{0.4 \text{ eV-s}}\right)^{\frac{1}{2}} \left(\frac{V_{\text{rf}} \cos\phi_0}{2.78 \text{ [MV]} \cos 37.6^\circ}\right)^{-\frac{1}{4}} \ . \tag{5.4}$$

The proposed tune is $\nu = 22.42$, giving $1/|\sin\mu/2| = 1.032$. In order that the nonadiabatic increase in transverse phase space is less than 1%, the minimum number of turns for pulsing the quadrupole should be 40 or for a minimum pulsing interval of 0.44 ms. This gives $\dot{\gamma} = 2260 \text{ sec}^{-1}$. If a jump of $\Delta\gamma_T = 1.5$ is preferred, $X_p^f - X_p^i = 5.88$ m and $\delta = 5.42 \times 10^{-3}$. The minimum number of turns becomes 44 or 0.49 ms, corresponding to $\dot{\gamma} = 2050 \text{ sec}^{-1}$. Note that if the tune were far away from an half-integer, the minimum number of turns would become much bigger.

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2. S.D. Holmes, *Main Injector Transition Jump*, Fermilab Internal Report MI-0008, 1989.